CURRENT ELECTRICITY

ALL DERIVATIONS

DRIFT VELOCITY

We may define **drift velocity** as the average velocity with which electrons get drifted towards the positive terminal of the battery under the influence of an external electric field.

Let the initial velocities of electrons (in the absence of battery) be $u_1, u_2, u_3 \ldots\ldots\ldots\ldots u_n$, then, $\dfrac{u_1 + u_2 + u_3 \ldots\ldots\ldots\ldots + u_n}{n} = 0$.

When the battery is applied, acceleration of each electrons is $a = \dfrac{eE}{m}$. When electrons move in a conductor, they keep colliding with the heavy ions present in it and come to a momentary rest. Time gap between two successive collisions is called relaxation time (τ).

Thus, if $v_1, v_2 \ldots\ldots\ldots v_n$ be the final velocities of electrons then, by definition, drift velocity is

$$v_d = \frac{v_1 + v_2 \ldots\ldots\ldots + v_n}{n}.$$

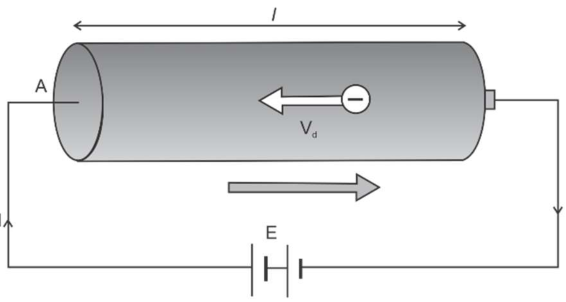
Since, $v_1 = u_1 + a\tau_1,\ v_2 = u_2 + a\tau_2,\ v_3 = u_3 + a\tau_3 \ldots\ldots\ldots v_n = u_n + a\tau_n$. Therefore v_d becomes

$$v_d = \frac{(u_1 + a\tau_1) + (u_2 + a\tau_2) + (u_3 + a\tau_3) \ldots\ldots\ldots + (u_n + a\tau_n)}{n}$$

$$\Rightarrow v_d = \left(\frac{u_1 + u_2 \ldots\ldots\ldots + u_n}{n}\right) + a\left(\frac{\tau_1 + \tau_2 \ldots\ldots\ldots\ldots + \tau_n}{n}\right)$$

Or $\boxed{v_d = \dfrac{eE}{m}\tau}$, where τ is average relaxation time.

RELATION BETWEEN CURRENT AND DRIFT VELOCITY



Consider a conductor of length ℓ and area of cross section A connected to battery of potential difference V. Then, volume of the conductor is $A\ell$. If number density of electrons in the conductor (number of electrons per unit volume) is n, then total number of electrons in conductor is $A\ell n$. Hence, total charge is, $q = A\ell ne$. Therefore, current in the conductor is given by $I = \frac{q}{t} \Rightarrow I = \frac{A\ell ne}{\left(\frac{\ell}{v_d}\right)}$.

or $\boxed{I = Anev_d}$.

PROOF OF OHM'S LAW AND FORMULA FOR RESISTANCE:

$\because I = Anev_d$ and $v_d = \frac{eE}{m}\tau$

$\therefore I = Ane\left(\frac{eE}{m}\tau\right)$

$\Rightarrow I = \frac{Ane^2E}{m}\tau$

$\Rightarrow I = \frac{Ane^2}{m}\left(\frac{V}{\ell}\right)\tau$

$\Rightarrow V = \frac{m\ell}{Ane^2\tau}I$

If physical conditions are constant $\frac{m\ell}{Ane^2\tau}$ is constant. Therefore, $V \propto I$.

Comparing (i) and (ii), we get $\boxed{R = \frac{m\ell}{Ane^2\tau}}$

Microscopic or vector form of ohm's law.

$\because J = \frac{I}{A}$

$\therefore J = \frac{Anev_d}{A} \Rightarrow J = ne\left(\frac{eE}{m}\tau\right)$

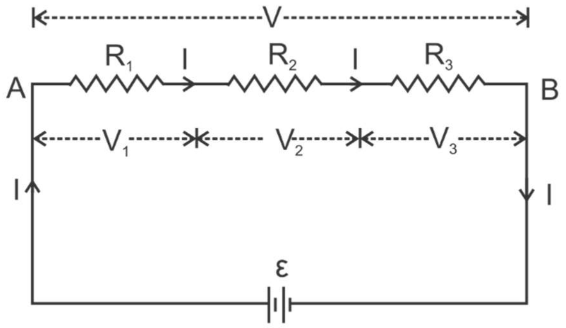
$\Rightarrow J = \frac{ne^2\tau}{m}E$

or $\boxed{\vec{J} = \sigma\vec{E}}$

COMBINATION OF RESISTORS (NOT IN SYLLABUS FOR SESSION 2022-23)

Series Combination

Consider two resistors R_1 and R_2 in series. The charge which leaves R_1 must be entering R_2.



Since current measures the rate of flow of charge, this means that the same current I flows through R_1 and R_2 . By Ohm's law:

Potential difference across $R_1 = V_1 = IR_1$, and

Potential difference across $R_2 = V_2 = IR_2$.

Potential difference across $R_3 = V_3 = IR_3$

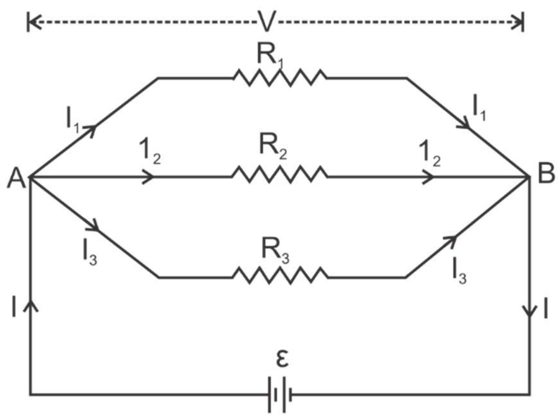
The potential difference V across the combination is $V_1 + V_2 + V_3$. Hence, $V = V_1 + V_2 + V_3 = I(R_1 + R_2 + R_3)$ This is as if the combination had an equivalent resistance R_{eq}, which by Ohm's law is

$$\boxed{R_{eq} = R_1 + R_2 + R_3}$$

This obviously can be extended to a series combination of any number n of resistors $R_1, R_2 ...., R_n$. The equivalent resistance R_{eq} is

$$\boxed{R_{eq} = R_1 + R_2 + R_3 .... + R_n}$$

Parallel combination.



The currents I, I_1, I_2 and I_3 shown in the figure are the rates of flow of charge at the points indicated. Hence,

$I = I_1 + I_2 + I_3$

The potential difference between A and B is given by the Ohm's law applied to R_1

$V = I_1R_1$

Also, Ohm's law applied to R_2 and R_3 gives

$V = I_2R_2$, $V_3 = IR_3$

$$\therefore I = I_1 + I_2 + I_3$$

$$\Rightarrow \frac{V}{R_{eq}} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

Or $\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$

If n resistors are connected in parallel, then,

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \ldots\ldots\ldots\ldots\ldots + \frac{1}{R_n}}$$

RELATION BETWEEN INTERNAL RESISTANCE, TERMINAL POTENTIAL DIFFERENCE AND EMF

Let ε be emf of the cell, V be the terminal potential difference, r be the internal resistance, R be external resistance and I be the current flowing in the circuit then, potential drop across internal resistance is Ir. Therefore, potential drop across external resistance is,

$$\boxed{V = \varepsilon - IR}$$

$$\Rightarrow Ir = \varepsilon - V$$

$$\Rightarrow r = \frac{\varepsilon - V}{I}$$

$$\Rightarrow r = \frac{\varepsilon - V}{\frac{V}{R}}$$

$$\Rightarrow r = \left(\frac{\varepsilon - V}{V}\right) \times R$$

Or $\boxed{r = \left(\frac{\varepsilon}{V} - 1\right) \times R}$

Charging. During charging of a cell, current flows in reverse direction with the help of external agency, so the terminal potential difference becomes $\boxed{V = \varepsilon + IR}$

COMBINATION OF CELLS

Like resistors, cells can also be connected in series and parallel combination.

Series combination. Consider two cells of emfs ε_1 and ε_2 and internal resistances r_1 and r_2 are connected in series.

If V_1 and V_2 be the terminal potential differences of the two cells, then V = $V_1 + V_2$

$$\Rightarrow V = (\varepsilon_1 - Ir_1) + (\varepsilon_2 - Ir_2)$$
$$\Rightarrow V = (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2)$$

Comparing this with $V = \varepsilon_{eq} - Ir_{eq}$ we get

$$\boxed{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2}$$

This result can be extended to series combination of n cells as

$$\boxed{\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 ....\varepsilon_n}$$

Parallel combination

If two cells are connected in parallel, terminal potential difference across them is same but current is different, ∴ total current

$$I = I_1 + I_2$$
$$\Rightarrow I = \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_1 - V}{r_2}$$
$$\Rightarrow I = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right)$$
$$\Rightarrow V\left(\frac{r_1 + r_2}{r_1 r_2}\right) = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 r_2} - I$$
$$\Rightarrow V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I\left(\frac{r_1 r_2}{r_1 + r_2}\right)$$

Comparing this with $V = \varepsilon_{eq} - Ir_{eq}$ we get

$$\boxed{\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2}}$$

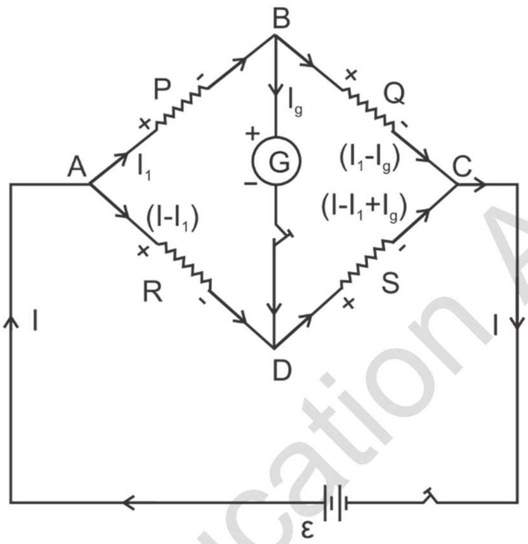
This result can be extended to parallel combination of n cells as

$$\varepsilon_{eq} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} + \frac{\varepsilon_3}{r_3} .... + \frac{\varepsilon_n}{r_n}$$

WHEATSTONE BRIDGE

Wheatstone bridge is a circuit which is used to measure accurately an unknown resistance.

Principle. It states that when the bridge is balanced (i.e. when $I_g = 0$), the product of resistances of opposite arms is equal.



. Applying Kirchhoff's second law to loop ABDA, we get

$$I_1P + I_gG - (I - I_1)R = 0$$

SInce $I_g = 0$

$$\therefore I_1P - (I - I_1)P = 0$$

$$\Rightarrow I_1P = (I - I_1)R \quad ....(i)$$

Applying second law in loop BCDB, we get

$$(I_1 - I_g)Q - (I - I_1 + I_g)S - I_gG = 0$$

$$\because I_g = 0$$

$$\therefore I_1Q - (I - I_1)S = 0$$

$$\Rightarrow I_1Q = (I - I_1)S \quad ....(ii)$$

From (i) and (ii) we get

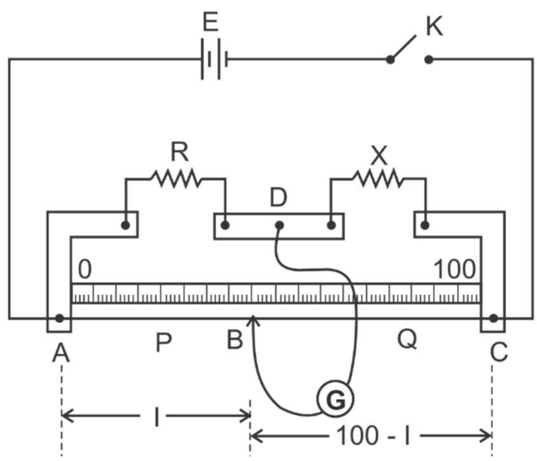
$$\boxed{\frac{P}{Q} = \frac{R}{S}}$$

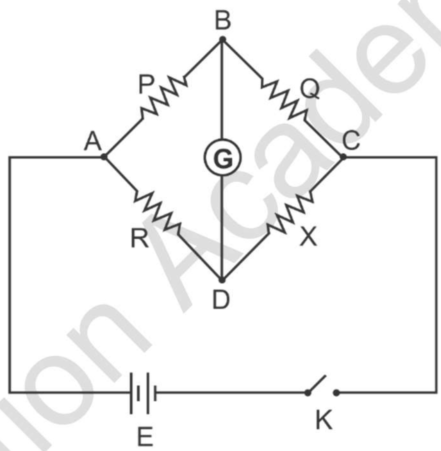
Or $\boxed{PS = QR}$

FINDING UNKNOWN RESISTANCE USING SLIDE WIRE BRIDGE (NOT IN SYLLABUS FOR SESSION 2022-23)

It a practical form of a Wheatstone bridge which is used to find an unknown resistance. Its operation is based on the principle of wheat stone bridge.

As shown in the figure introduce a suitable value of R and close key K. Move the jockey on the wire AC to obtain the null point (i.e. zero reading of the galvanometer). Let point B be the null point on the wire AC. Let length AB be ℓ, therefore length BC is $100-\ell$. As the bridge is balanced, therefore, by Wheatstone bridge principle, we have





$$\frac{P}{Q}=\frac{R}{S}$$

If r be the resistance per cm length of the wire, then

P = resistance of length ℓ of the wire = ℓr

Q = resistance of length $100-\ell$ of the wire = $(100-\ell)r$

$$\therefore \frac{\ell r}{(100-\ell)r}=\frac{R}{S}$$

Or $$\boxed{S=\left(\frac{100-\ell}{\ell}\right)\times R}$$

Knowing ℓ and R, S can be determined.

PROOF OF WORKING PRINCIPLE OF POTENTIOMETER (NOT IN SYLLABUS FOR SESSION 2022-23)

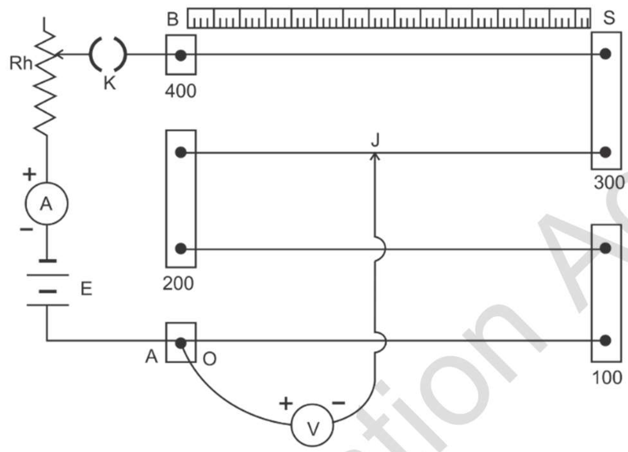
Principle. The working of potentiometer is based on the fact that the fall of potential across any portion of the wire is directly proportional to the length of that portion provided the wire is of uniform area of cross section and a constant current is flowing through it.

Proof. Let A be the area of cross section, ρ be the resistivity of the material pf the wire, V be potential difference across length ℓ whose resistance is R. Let I be the current flowing through the wire, then by Ohm's law

$$V = IR$$

$$\text{As } R = \rho\frac{\ell}{A}$$

$$\text{we have } V = I\rho\frac{\ell}{A}$$

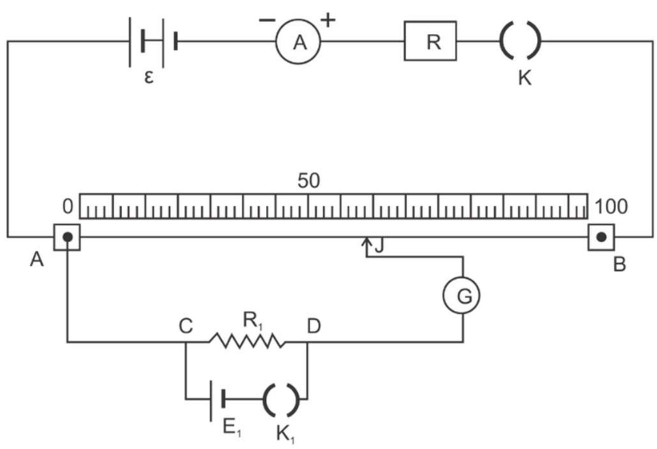


$$\text{or } V = \left(\frac{I\rho}{A}\right)\ell$$

$$\text{or } V \propto \ell$$

$\frac{V}{\ell}$ is called potential gradient of the wire i.e. fall in potential per unit length of the wire.

DETERMINING A POTENTIAL DIFFERENCE USING POTENTIOMETER (NOT IN SYLLABUS FOR SESSION 2022-23)



Close key K and adjust the value of R so that fall of potential across the potentiometer wire is greater than the potential difference to be measured. Close key K_1. Adjust the position of jockey on potentiometer wire where is pressed, the galvanometer shows no deflection. Let that position be J. Let length AJ be ℓ. If k is the potential gradient of potentiometer wire, then potential difference across R_1 i.e.

$$V = k\ell$$

If r is the resistance of potentiometer wire of length L, then current through potentiometer wire is ,

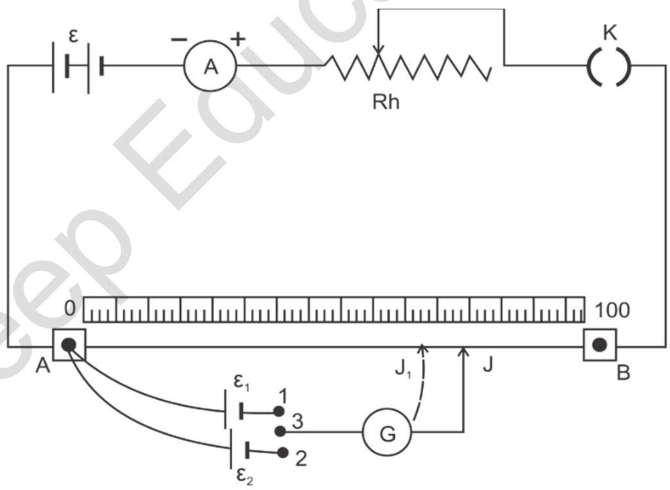
$$I = \frac{\varepsilon}{R+r}$$

Potential drop across potentiometer wire = Ir = $\left(\frac{\varepsilon}{R+r}\right)r$

Potential gradient of potentiometer wire $k = \left(\frac{\varepsilon}{R+r}\right)\frac{r}{L}$

$$\therefore \boxed{V = \left(\frac{\varepsilon}{R+r}\right)\frac{r}{L} \times \ell}$$

COMPARING EMFS OF TWO CELLS USING POTENTIOMETER (NOT IN SYLLABUS FOR SESSION 2022-23)



Two cells whose emfs are to compared are connected as shown in the figure. First connect terminal 1 with terminal 3 such that cell with emf ε_1 comes in the circuit. If ℓ_1 is the balancing length in this case, we can write

$$\varepsilon_1 = k\ell_1 \qquad \text{..........(i)}$$

Now disconnect 1 and 3 and connect 2 and 3. Now cell with emf ε_2 comes in the circuit. If ℓ_2 is the balancing length in this case, then

$$\varepsilon_2 = k\ell_2 \qquad \text{..........(ii)}$$

From (i) and (ii) we get $\boxed{\dfrac{\varepsilon_1}{\varepsilon_2} = \dfrac{\ell_1}{\ell_2}}$

DETERMINING INTERNAL RESISTANCE OF A CELL (NOT IN SYLLABUS FOR SESSION 2022-23)

Close key K and note the balancing length. Let it be ℓ_1. Now, emf of the cell, ε = potential difference across length ℓ_1 of the potentiometer wire

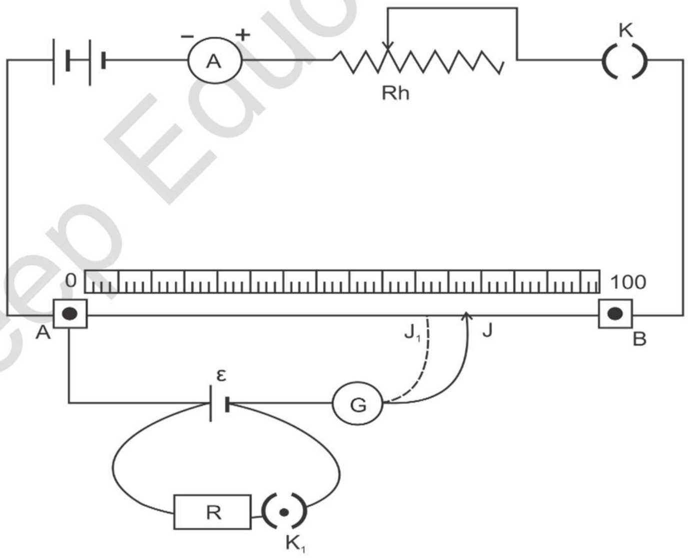
Or $\varepsilon = k\ell_1$

Now close key K_1 so that the resistance R is introduced in the circuit. Again, find the position of null point. Let balancing length in this case be ℓ_2. Then, potential difference between two terminals of the cell, V = potential difference across length ℓ_2 of the potentiometer wire

i.e. $V = k\ell_2$

$$\frac{\varepsilon}{V} = \frac{\ell_1}{\ell_2}$$

$$\therefore \quad \frac{\varepsilon}{V} = \frac{\ell_1}{\ell_2}$$



$$\because r = \left(\frac{\varepsilon}{V} - 1\right) \times R \ \text{v}$$

$$\therefore \boxed{r = \left(\frac{\ell_1}{\ell_2} - 1\right) \times R}$$

Knowing the values of ℓ_1, ℓ_2 and R, internal resistance of the cell can be determined.

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